GE2MI conference on PDE's, Control Theory and Related Topics

From September 20 to September 23, 2018

Organizers: Oana Ivanovici, Gilles Lebeau, Pedro Freitas, Teresa Monteiro Fernandes

ABSTRACTS

- **Karine Beauchard** *Title: Null controllability of hypoelliptic equations*

  Abstract: We study the geometric control condition for degenerate parabolic equations, of hypoelliptic type, posed on the whole space. For Ornstein-Uhlenbeck equations, this question is related to a spectral Inequality for the Fourier transform due to Logvinenko, Sereda and Kovrijkine. For quadratic equations with zero singular space, this question is related to a spectral inequality for finite sums of Hermite functions.

- **Anne Boutet de Monvel** *Title: The 1D focusing NLS equation: long-time dynamics of step-like oscillating data*

  Abstract: In this talk I will consider a solution $q(x,t)$ of the 1D focusing nonlinear Schrodinger equation $i\partial_t q + \partial_{xx}^2 q + 2|q|2q = 0$ whose initial values $q(x,0)$ approach two different plane waves at minus and plus infinity. The goal is to determine the long-time asymptotic behaviour of this solution. I will present several results, depending on the relative values of the parameters. The analysis is based on the Riemann Hilbert approach which is a special formulation of the inverse scattering transform method.

- **S Correia**

  Title: Self-similar solutions for the modified Korteweg-de Vries equation

  Abstract: We consider the modified Korteweg-de Vries equation over $\mathbb{R}$, $\partial_t u + \partial_{xxx}^3 u = u^3$. This equation arises, for example, in the theory of water waves and vortex fil- aments in fluid dynamics. A particular class of solutions to (mKdV) are those which do not change under scaling transformations, the so-called self-similar so- lutions. Self-similar solutions blow-up when t goes to 0 and determine the asymptotic behaviour of the evolution problem. First, we present the construction of self-similar solutions through a fixed-point argument on the Fourier space and by means of stationary phase analysis. This structure is in contrast with the usual shooting methods developed to find self- similar solutions. We determine almost explicitly the behaviour on the frequency side, which is essential for the subsequent dynamical analysis. In the second part, we show existence and uniqueness of small solutions to the (mKdV) lying on a critical space which includes both regular and self-similar solutions. In this context, the standard fixed-point arguments fail and a new argument has to be found. Finally, the asymptotic stability of self-similar solutions is proven. This is joint work with Raphael Côte and Luis Vega.

- **J.Paulo Dias**

  Title: A coupled system of a complex Ginzburg-Landau equation with a quasilinear conservation law

  Abstract: Talk based on a joint work with Filipe Oliveira and Hugo Tavares. We study a coupled system of a complex time dependent Ginzburg-Landau equation with a quasilinear conservation law,which can describe the interaction between a laser beam and a fluid flow. We prove the existence of a local in time strong solution for the associated Cauchy problem and, for a certain class of flux functions, the existence of global weak solutions. Furthermore we prove the existence of standing waves in several cases.

- **Armand Koenig**

  Title: (Non) Null Controllability of the Fractional Heat Equation and of Related Equations

  Abstract: The heat equation on bounded domain is null-controllable in arbitrarily small time and with an arbitrarily small control domain, it is now well-known. But if we replace the laplacian in the heat equation by a degenerate elliptic operator, the picture changes drastically: the null-controllability can depend on time and the control domain. We will look at models of such equations, namely the fractional heat equation, and we will deduce some information on actual degenerate parabolic equations.

- **Richard Lascar**

  Title: Dispersive Estimates for waves in a general strictly convex domain

  Abstract: We prove here dispersive estimates for waves in general convex domains; it exhibits a 1/4 loss power $(h/t)^{1/4}$ with respect to the boundary less case. A slight refinement of this estimate is in fact optimal, because of appearance of swallowtail singularities in the wave front at arbitrary small times.?The key tool of the proof is a parametrix construction involving Airy functions, described as a superposition of waves or as a sum over gallery modes according the distance of the source from the boundary. These two aspects are correlated trough a variation of Poisson formula we dub Airy-Poisson summation formula.As a corollary one obtains Strichartz estimates with a free dimensional 1/4 loss with respect to the boundary less case, but it will be improved by the results from the talk of F.Planchon. This is joint work with O. Ivanovici, G.Lebeau, and F. Planchon

- **Pierre Lissy**

  Title: Approximation of some control problems by finite-difference method

  Abstract : In this talk, we will explain some difficulties that may arise when one tries to approximate a control problem, notably the loss of uniform controllability (with respect to the mesh-size) that prevents from recovering a control for the continuous problem as a limit of controls for the discrete problem. As an example, we consider a finite-difference semi-discrete scheme for the approximation of internal controls of a one-dimensional wave (or fractional wave-like) equations). The continuous problem is controllable. However, the high frequency numerical spurious oscillations lead to a loss of the uniform controllability property. We will detail how to restore the uniform controllability property thanks to an appropriate filtering method on the initial condition. The proof is mainly based on the moment method.

- **Frédéric Marbach**

Title: Quadratic behavior for the small-time local controllability of scalar-input parabolic equations

Abstract: In this joint work with Karine Beauchard, we investigate the controllability of nonlinear scalar-input parabolic systems. When the linearized system is not controllable, we introduce a systematic method to study the second-order approximation of the system. We exhibit systems for which this method either leads to negative or positive results, leading to a panorama of possible behaviours.

- **Ivan Moyano**

  Title : Stability estimates in Wasserstein distance for some kinetic equations

  Abstract : In this talk I intend to illustrate the use of Wasserstein distances in the space of probability measures, arising in the context of optimal transportation, in order to obtain stability estimates for some kinetic equations. We shall first recall how this approach allowed to obtain a satisfactory uniqueness result for the weak solutions of the Vlasov-Poisson system (G.Loeper, 2005). In a second time we explain how to adapt these ideas in the context of the Vlasov-Navier-Stokes system (D.Han-Kwan, E.Miot, A.Moussa and I.M., 2017), which requires to suitably modify the original approach. Finally, we give some highlights on how to develop this approach on a collisional regime, which requires the use of stochastic characteristics.

- **F. Planchon**

  Title: Strichartz estimates for the wave equation in strictly convex domains

  Abstract: We prove sharper Strichartz estimates than expected from the optimal dispersion bounds. This follows from taking full advantage of the space-time localization of caustics. Several improvements on the parametrix construction are obtained along the way and are of independent interest. Moreover, we extend the range of known counterexamples by propagating carefully constructed Gaussian beams, proving that our Strichartz estimates are sharp in some regions of phase space. This is joint work with O.Ivanovici and G.Lebeau.

- **Marjolaine Puel**

  Title: Fractional diffusion for the Fokker Planck equation with heavy tail equilibria

  Abstract: Diffusion approximation is a well known process to approximate kinetic equations by macroscopic equations in order for examples to reduce the number of variables for numerical purpose. The goal of this talk is to give a overview of the recent progresses in diffusion approximation and to focus on the special case of the Fokker Planck equation with heavy tail equilibrium that model the cooling of atoms. In this particular case, the difficulty is due to the absence of a spectral gap for the collision operator. The one dimensional case is is a joint work with Gilles Lebeau. What about the multi-D case ?

- **Hugo Tavares**

  Title : Some aspects about weakly coupled elliptic systems: existence results and phase separation

  Abstract : In this talk, we will deal with systems of stationary reaction-diffusion equations where the interaction between different components is either cooperative or competitive. Our aim will be to explain some of the relevant questions that can be asked for each type of interaction, explaining as well the motivations for its study. We will survey some of the results

proved in the last few years, discussing in general the existence and characterisation of positive solutions. Furthermore, we will explain how a strong competition induces a phase separation phenomenon and gives rise to a free boundary problem. In the last part of the talk, we will consider nonlocal interaction terms between the components. We will highlight some of the similarities and differences between the local and the nonlocal cases, showing some recent results in the nonlocal one.

- **Nikolay Tzvetkov**

  Title: On the transport of Gaussian measures under the flow of Hamiltonian partial differential equations

  Abstract : We will consider the fourth order Nonlinear Schrodinger equation, posed on the circle, with initial data distributed according to the white noise. This problem is well posed for smooth initial data. It is therefore natural to consider the sequence of smooth solutions with data distributed according regularisations (by convolution) of the white noise. We show that a renormalisation of this sequence converges to a unique limit. The limit has the white noise as an invariant measure. The proof shares some features with the modified scattering theory which received a lot of attention in the PDE community. As a consequence the solution has a more intricate singular part compared to the large body of literature on probabilistic well-posedness for dispersive PDE's. This is a joint work with Tadahiro Oh and Yuzhao Wang.

- **Chenmin Sun**

  Title: Stabilization of KP-II equation

  Abstract: KP-II equation is a dispersive equation modeling the propagation of shallow water waves in bidimensional channels. In this talk, I will present a stabilization result for KP-II equation with ?arbitrarily large initial data in $H^s(\mathbb{T}^2)$. More precisly, when the damping is added on a vertical strip, the solution of damped KP-II equation decays exponentially fast as $t \to +\infty$, provided that the initial data satisfies a partial compactness assumption. The proof combines the technique of nonlinear dispersive PDEs and a simple one dimensional semi-classical analysis for the linearized KP-II equation.

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