Hamiltonian PDEs and nonlinear waves
La Thuile, February 10th-16th, 2019

This conference is supported by the ERC grant ANADEL 757 996, by the ERC grant SINGWAVES 646 650 and the University of Pisa.

1 Schedule

	Monday	**Tuesday**	**Wednesday**†	**Thursday**	**Friday**
9h⋆–10h15	Faou I	Michel	Schlein II	Franzoi	Montalto
10h15–10h45	Coffee break				
10h45–12h	Schlein I	Faou II	Stolovitch	Alazard I	Alazard II
12h-14h	Lunch				
14h–16h	Free discussions				
17h10–18h	Cossetti	Pacherie	Excursion	Banica	Ingremeau
18h10-19h	Grébert	Iandoli		Iooss	Kappeler

⋆: 9h15 from Tuesday, †: Wednesday morning: 9h15-10h30, 10h30-11h, 11h-12h.

2 Mini-courses

Erwan Faou: Wave turbulence: Linearized kinetic theory unchained

We consider the problem of wave turbulence, i.e. a system of semi-linear wave with random initial condition and continuous limit of frequency. The main example will be stochastic and deterministic semi-linear systems with bounded and almost continuous set of frequencies. Such systems can be obtained by considering nonlinear lattice dynamics or truncated partial differential equations on large periodic domains. We assume that the nonlinearity is small and that the noise is small or void and acting only in the angles of the Fourier modes (random phase forcing). We consider random initial data and assume that these systems possess natural invariant distributions corresponding to some Rayleigh-Jeans stationary solutions of the wave kinetic equation appearing in wave turbulence theory. We consider random initial modes drawn with probability laws that are perturbations of theses invariant distributions. In the stochastic case, we prove that in the asymptotic limit (small nonlinearity, continuous set of frequency and small noise), the renormalized fluctuations of the amplitudes of the Fourier modes converge in a weak sense towards the solution of the linearized wave kinetic equation around these Rayleigh-Jeans spectra. Moreover, we show that in absence of noise, the deterministic equation with the same random initial condition satisfies a generic Birkhoff reduction in a probabilistic sense, without kinetic description at least in some regime of parameters.

Benjamin Schlein: Excitation spectrum for dilute Bose-Einstein condensates

We consider systems of N bosons confined in a box with volume one and interacting through a potential with scattering length of the order $1/N$ (Gross-Pitaevskii regime). We determine the low-energy spectrum, i.e. the ground state energy and low-lying excitations, up to errors that vanishes in the limit of large N, confirming the validity of Bogoliubov's theory.

Thomas Alazard: A Morawetz inequality for water waves

We consider gravity water waves in two space dimensions. Assuming uniform energy bounds for the solutions, we prove local energy decay estimates. Our result is uniform in the infinite depth limit. This is a joint work with Mihaela Ifrim and Daniel Tataru.

3 Talks

Lucrezia Cossetti: Multipliers method for Spectral Theory

Originally arisen to understand characterizing properties connected with dispersive phenomena, in the last decades the multipliers method has been recognized as a useful tool in Spectral Theory, in particular in connection with proof of absence of point spectrum for both self-adjoint and non self-adjoint operators.

Starting from recovering very well known facts about the spectrum of the free Laplacian $H_0 = -\Delta$ in $L^2(\mathbb{R}^d)$, we will see the developments of the method reviewing some recent results concerning self-adjoint and non self-adjoint perturbations of this Hamiltonian in different settings, specifically both when the configuration space is the whole Euclidean space $\mathbb{R}^d$ and when we restrict to domains with boundary. We will show how this technique allows to detect physically natural repulsive and smallness conditions on the potentials which guarantee the absence of eigenvalues. Some very recent results concerning Pauli operators will be also presented.

The talk is based on joint works with L. Fanelli and D. Krejčiřík.

Benoît Grébert : Rational normal forms: the Killbill theory.

We consider general classes of nonlinear Schrödinger equations on the circle with nontrivial cubic part and without external parameters. We construct a new type of normal forms, namely rational normal forms, on open sets surrounding the origin in high Sobolev regularity. With this new tool we prove that, given a large constant M and a sufficiently small parameter ε, for generic initial data of size ε, the flow is conjugated to an integrable flow up to an arbitrary small remainder of order ε^{M+1}. This implies that for such initial data $u(0)$ we control the Sobolev norm of the solution $u(t)$ for time of order ε^{-M}. Furthermore this property is locally stable: if $v(0)$ is sufficiently close to $u(0)$ (of order $\varepsilon^{3/2}$) then the solution $v(t)$ is also controled for time of order ε^{-M}. (Joint work with Erwan Faou and Joackim Bernier)

Laurent Michel: Small eigenvalues of Witten Laplacian: old and new

The Witten Laplacian was introduced by Witten to give an analytical proof of Morse inequalities. It plays also a crucial role in the analysis of metastable processes such as Langevin equation for its small eigenvalues yield the relevant time scales of the return to equilibrium. In this talk we will explain old and new results on the asymptotic behavior of this eigenvalues at low temperature.

Eliot Pacherie: Properties of some travelling waves for the Gross-Pitaevskii equation

The Gross-Pitaevskii equation is a nonlinear Schrödinger equation with a nonzero boundary condition at infinity. Among other knowns propreties, it exhibits some particular stationnary solutions called vortices. In this talk, we will present a method to construct travelling waves solutions as a perturbation of two well separated vortices, and then use this particular solution to give some coercivity results for these travelling waves.

Felice Iandoli: Long time solutions for fully non-linear Schrödinger equations on the circle.

I will discuss the local in time well posedness for a large class of fully-nonlinear Schrödinger equations on the circle. After that I will give some non trivial estimates on the time of existence of the solutions in the case of small initial data. This is a joint work with Roberto Feola.

Laurent Stolovitch: Linearization of neighborhoods of embeddings of complex compact manifolds

In this work, we address the following question due to Grauert: if a neighbourhood of an holomorphically embedded complex compact manifold is formally equivalent to another one, are they biholomorphically equivalent ? We shall present the case where one of ambient manifold M the compact manifold C is embeded into is the normal bundle N_C of C into another M^*. The solution to this problem involves "small divisors problems". This is joint work with X. Gong.

Luca Franzoi: Fast oscillating forced systems: reducibility for a linear Klein-Gordon equation

Aim of this talk is to show a reducibility result for a linear Klein-Gordon equation on a one dimensional compact interval, whose dynamics is subject to an external forcing potential that depends quasiperiodically on time. The size of the latter is not necessarily small, and so this is not a perturbative result. Nevertheless, reducibility is achieved by taking the frequency vector of the potential sufficiently large. In the first half of the talk I will briefly review some features of fast driven systems, Floquet theory in infinite dimension and the problem of the growth of the Sobolev norms, in order to present the main result and its corollaries. In the second half I will focus on the two main novel points of the proof: the use of pseudodifferential operators for performing one step of Magnus normal form, which recovers a perturbative setting; the structure of the nonresonant conditions needed for closing our subsequent KAM reduction scheme. If time permits, I will say some words on what happens when one considers different equations, rather than the Klein-Gordon one. This is a joint work with Alberto Maspero.

Valeria Banica: Evolution of polygonal lines by the binormal flow

The binormal flow is a 3-D curve evolution equation that models the dynamics of filament vortices in 3-D fluids. The first part of this talk will be dedicated to the presentation of the classical connection established between the binormal flow and the 1-D cubic Schrödinger equation. We shall see in particular that the 1-D cubic Schrödinger equation with initial data a Dirac mass is related to the formation in finite time of a corner by the binormal flow. As a first result we construct solutions of the 1-D cubic Schrödinger equation in link with initial data a sum of several Dirac masses. Then we shall construct and describe a new class of singular solutions of the binormal flow, that in particular we continue after the time of singularity formation. This is a joint work with Luis Vega.

Gérard Iooss: Existence of quasipatterns in the superposition of two hexagonal periodic patterns

Let us consider a quasilattice, spanned by the superposition of two hexagonal lattices in the plane, differing by a rotation of angle ß. We study bifurcating quasipatterns solutions of the Swift-Hohenberg PDE, built on such a quasilattice, invariant under rotations of angle π /3. For nearly all ß, This is a small divisor problem. We prove that in addition to the classical hexagonal patterns, there exist two branches of bifurcating quasipatterns, with equal amplitudes on each basic lattice.

Riccardo Montalto: Quasi-periodic solutions of pure gravity water waves in finite depth

In this talk I will present some recent results concerning the existence and the stability of quasi-periodic solutions for the 2D-Euler equation of an irrotational and incompressible fluid under the action of the gravity, in finite depth. The main difficulties are the fully non-linear nature of the gravity water waves equations (the highest order x-derivative appears in the nonlinear term but not in the linearization at the origin) and the fact that the linear frequencies grow only in a sublinear way at infinity. The proof is obtained by combining perturbation theory and Pseudo differential calculus, to solve the linearized problem at each step of a Nash-Moser iteration.

Maxime Ingremeau: Quantum ergodicity on large quantum graphs

Metric graphs (also known as quantum graphs) have been studied by physicists and mathematicians for a long time, since they are a good toy model to understand quantum chaos. In particular, much attention has been devoted to study the asymptotic behaviour of eigenvalues and eigenfunctions on a fixed quantum graph, when the frequency goes to infinity. In this talk, we will study another interesting regime : when the frequency is fixed, but the graph becomes larger, and more and more complicated. In particular, we will show a quantum ergodicity result for large quantum graphs which are expanders and locally look like trees. This is joint work with Nalini Anantharaman, Mostafa Sabri and Brian Winn.

Thomas Kappeler: Normal form coordinates for the KdV equation having expansions in terms of pseudodifferential operators

Complex normal coordinates for integrable PDEs on the torus can be viewed as 'nonlinear Fourier coefficients'. Near an arbitrary compact family of finite dimensional tori, left invariant under the KdV flow, we construct a real analytic, 'nonlinear Fourier transform' for the KdV equation having the following main properties:

(0) When restricted to the family of finite dimensional tori, the transformation coincides with the Birkhoff map.

(1) Up to a remainder term, which is smoothing to any given order, it is a pseudodifferential operator of order 0 in the normal directions with principal part given by the Fourier transform.

(2) It is canonical and the pullback of the KdV Hamiltonian is in normal form up to order three.

Such coordinates are a key ingredient for studying the stability of finite gap solutions of arbitrary size (periodic multisolitons) of the KdV equation under small, quasi-linear perturbations. This is joint work with Riccardo Montalto.