

Abstracts

Mini Courses

- **Massimiliano Berti** : Dynamics of water waves

In this mini-course I shall present recent results about the water waves equations of a bi-dimensional fluid under the action of gravity and capillary forces, with space periodic boundary conditions. We shall discuss both Birkhoff normal form results as well as existence of quasi-periodic solutions. Major difficulties are the quasilinear nature of the water waves equations.

- **Nicolas Burq** : Wave control and second-microlocalization on geodesics

For the wave equation on a compact manifold , in the case of a control localized by a continuous localization functions, the geometric control condition is necessary and sufficient for exact controlability. In this article, on surfaces, in the special case where $a(x) = \sum_{j=1}^N a_j 1_{x \in R_j}$ (R_j are geodesic polygons), we give a very natural necessary and sufficient geometric condition. The proof requires understanding the conormal concentration properties of quasi-modes on geodesics (second microlocalization). In the course, I will present in a first part the classical control results and in a second part the second microlocalization procedure.

- **Joachim Krieger** : On stability of type II blow up solutions for certain critical nonlinear wave equations

I will discuss a recent result, joint with S. Burzio, on co-dimension one stability of type II blow up solutions constructed with Schlag and Tataru for the critical focussing wave equation $\square u = -u^5$ in 3D. I will also discuss a prospective result (current work in progress), joint with S. Miao, on stability of blow up solutions for the critical co-rotational Wave Maps problem.

- **Anne-Sophie De Suzzoni** : Weak turbulence

We will present the notion of weak turbulence and kinetic equations related to weak turbulence. Their object of study is the correlations between Fourier modes of a solution to some Hamiltonian equation and their evolution in time in a large box and weak interaction limit. The solution is random not in the sense that the equation is random but because it is taken initially random. There will be two parts : in the first

one, we will introduce the notion of random phase approximation and give a result on the Benjamin-Bona-Mahony and Kadomtsev-Petviashvili equations when the initial datum is a Gaussian field. In the second one, we will take an initial datum whose law is invariant under the action of the flow of the cubic Schrödinger equation and study the correlations between Fourier mode at time t and at time 0 . We shall observe at the level of formal computations some dissipation. Joint work with : Nikolay Tzvetkov, Zaher Hany.

Talks

- **Renato Luca** : Invariant measures for the periodic derivative nonlinear Schrödinger equation

In this talk we will consider the periodic (one dimensional) derivative nonlinear Schrödinger equation with small L^2 data. We will construct an infinite sequence of invariant measures associated to the integrals of motion of the equation. This is a joint work with Giuseppe Genovese and Daniele Valeri.

- **Ludovick Gagnon** :Sufficient conditions for the controllability of wave equations with a transmission condition

We consider waves propagating in two medium with different constant speed of propagation. At the interface between the two medium, a transmission condition is imposed, equivalent of the Snell-Descartes law. Under geometrical hypothesis, we present sufficient geometrical conditions for the controllability of this equation.

- **Michela Procesi** : Small quasi periodic solutions for a class of quasi-linear dispersive PDEs on the circle

I shall discuss existence and linear stability of small quasi-periodic solutions for quasi linear dispersive PDEs on the circle; I shall particularly concentrate on the DP equation which is an integrable Hamiltonian system with asymptotically linear dispersion law. I will give an overview of the general strategies as well as of the difficulties in dealing specifically with the DP equation. This is based on joint work with R. Feola and F. Giuliani.

- **David Lafontaine** : Strichartz estimates without loss outside two strictly convex obstacles

We are concerned by waves and Schrödinger equations outside obstacles. In order to study the perturbative theory and the non-linear equations associated to these equations, it is crucial to understand how the linear flow decays. Two tools permit in particular to quantify such a decay: estimates of the local energy, and the so-called Strichartz estimates, which control the space-time norms of solutions. When a trapped ray exists, a loss occurs with respect to the free space in the local energy decay. In

contrast, we will show global Strichartz estimates without loss in a geometry with an unstable trapped ray: the exterior of two strictly convex obstacles.

- **Jacopo Bellazzini** : Long time dynamics for semirelativistic NLS and half wave equation

Aim of the talk is to present recent results concerning existence and dynamical properties of ground states for semirelativistic NLS and HW. Joint work with V. Georgiev and N. Visciglia.

- **Felice Iandoli** : Local well posedness for quasi-linear NLS with large Cauchy data on the circle

I discuss local in time well-posedness for a large class of quasi-linear Hamiltonian, or parity preserving, Schrödinger equation on the circle. Using para-differential tools I show that the system can be reduced to another one with symbols which, at the positive order, are constant and purely imaginary. This allows to obtain a priori energy estimates on the Sobolev norms of the solution. This is a joint work with Roberto Feola.

- **Joachim Bernier** : Quasi orbital stability of approximate solitons of DNLS

We study the existence and the orbital stability of solitons of the Discrete NonLinear Schrödinger equation near the continuous limit. With the Shannon interpolation, we get a natural continuous advection on sequences. We explain and quantify how the aliasing errors cause the default of invariance of DNLS by this advection. Consequently, we design a version of DNLS without aliasing and we construct solitons for this equation. With the energy-momentum method, we show that these solitons are orbitally stable for DNLS in times inversely proportional to the step size of the grid.

- **Lucrezia Cossetti** : Unique Continuation for the Z-K dispersive equation

In this talk we will analyse uniqueness properties of solutions of the Zakharov-Kuznetsov (Z-K) equation

$$\partial_t u + \partial_x^3 u + \partial_x \partial_y^2 u + u \partial_x u = 0, \quad (x, y) \in \mathbb{R}^2, \quad t \in [0, 1].$$

Mainly motivated by the very well known PDE's counterpart of the Hardy Uncertainty Principle we will provide a two times unique continuation result. More precisely we will prove that if the difference $u_1 - u_2$ of two solutions u_1, u_2 of this equation at two different times $t_0 = 0$ and $t_1 = 1$ decays fast enough, then $u_1 \equiv u_2$.

Moreover as we will see, the decay rate needed in order to obtain the stated uniqueness reflects, as expected, the asymptotic behavior of the fundamental solution of the associated linear problem.

The talk is based on a joint work with L. Fanelli ("Sapienza") and F. Linares (IMPA, Rio de Janeiro).

- **Dario Bambusi** : Growth of Sobolev norms in time dependent abstract Schrödinger equations

Joint work with Benoit Grébert, Alberto Maspero, Didier Robert .

I will present an abstract theorem giving a $\langle t \rangle^\epsilon$ bound ($\forall \epsilon > 0$) on the growth of the Sobolev norms in linear Schrödinger equations of the form $Im(\dot{\psi}) = H_0\psi + V(t)\psi$ when the time $t \rightarrow \infty$. The abstract theorem is applied to several cases, including the cases where (i) H_0 is the Laplace operator on a Zoll manifold and $V(t)$ a pseudodifferential operator of order smaller than 2; (ii) H_0 is the (resonant or non-resonant) Harmonic oscillator in \mathbb{R}^d and $V(t)$ a pseudodifferential operator of order smaller than H_0 depending in a quasiperiodic way on time. The proof is obtained by first conjugating the system to some normal form in which the perturbation is a smoothing operator and then applying some known results.

I will also present some counterexamples giving lower bounds to the Sobolev Norms.

- **Oana Ivanovici** : Dispersive estimates for the wave equation outside strictly convex obstacles : the 3D situation

We consider the linear wave equation outside a compact, strictly convex obstacle in R^3 with smooth boundary and we show that the linear wave flow satisfies the dispersive estimates as in R^3 (which is not necessarily the case in higher dimensions). This is joint work with Gilles Lebeau.