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DEPARTMENT OF MATHEMATICS

BERKELEY, CALIFORNIA 94720-3840
tataru@math.berkeley.edu

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Report on the Habilitation Mémoire of Oana Ivanovici

Oana Ivanovici's Habilitation dissertation provides a succinct but very clear and well prepared overview of her excellent and extensive contributions to the study of dispersive decay properties for the wave and Schrödinger equations in the context of domains with boundaries.

To provide a thorough evaluation of her work, it is useful to begin with a brief overview of the broader research area, namely dispersive equations. These are evolution equations which on one hand have wave-like solutions and conservative dynamics, while, on the other hand, have dispersive properties, in the sense that the group velocity depends on the frequency. The most significant examples include wave and Schrödinger evolutions, both of which arise in the description of uncountable physical phenomena.

To my mind, the fundamental questions in this research area are nonlinear. Understanding both local and long time dynamics in nonlinear dispersive equations hinges on the study of the balance of linear dispersion and nonlinear interactions. In this context, a key component of the analysis is to find good ways to measure the dispersive decay properties of linear flows. Here the linear equations of interest are often the linearizations around nonlinear solutions of interest. Oana's research has been primarily focused on the linear side of things; however, the results should always be though of in the broader nonlinear context.

Historically, two useful ways to measure dispersive decay have been dispersive estimates and Strichartz estimates. The dispersive estimates apply in the case when the initial data is localized, and then the goal is to measure the uniform decay properties of the evolution as a function of time. Heuristically, the intuition here is that waves with different frequencies will move in different directions. Strichartz estimates, on the other hand, apply in situations when the initial data is not localized, e.g. of L^2 type. Here uniform decay is out of the question by translation invariance, so the goal becomes, instead, to measure averaged decay.

Initially, both dispersive and Strichartz estimates were considered in the constant coefficient case. Later, the study of quasilinear problems led to the need to consider variable coefficient settings. Oana Ivanovici's work is devoted to the situation when not only the coefficients are variable, but also the domain under consideration has boundary, with suitable boundary conditions for the evolution. One may be tempted to think of this as very specialized, and qualify the scope of this work as narrow. However, that would be a mistake. Indeed, this problem is so complex so that addressing it requires a very broad array of ideas and techniques from microlocal analysis, harmonic analysis and even some number theory.

At the classical level, the decay properties of waves are closely related to the structure and dispersion of the Hamilton flow, which in both the wave and in the Schrödinger case means the structure of the geodesic flow, with reflections occurring where geodesics hit the boundary. Because of this, one can distinguish two fundamentally different situations, namely that of (geodesically) convex or concave domains. In the first case, one may have a very large number of reflections over a short time, and the problem can be primarily thought of as a local problem. In the second case, the reflection points are unique, and the problem can be thought of as a long time problem; a good model is then the exterior of a convex set. The work presented in the dissertation separately addresses the two cases.

Following the structure of the dissertation, let us begin our discussion with the convex case. Here the first objective of Oana's work has been to study the dispersive decay of the linear waves, which in essence means to understand the pointwise decay of the fundamental solution, i.e. the Green's function for either the wave or the Schröedinger flow. The difficulty is that repeated reflections immediately produce caustics, and one of the early successes was to both identify and accurately measure the worst case scenario, which turns out to be a swallow tail pattern. Together with the more classical gallery modes, this showed that there is indeed a loss in decay compared to the boundaryless case. In more recent work, all additional concentration scenarios were identified and measured, which essentially led to optimal dispersive decay bounds. Two difficulties here that should be mentioned are (i) at the level of the microlocal parametrices, which are quite complex both in terms of phases and amplitudes, and (ii) at the level of stationary phase arguments, where she identified two complementary but touching regimes, where the peaks given by stationary phase corresponding to different critical points are either isolated or overlapping. These difficulties are already present in the model case, the so-called Friedlander model, and it only gets harder in the general case.

The second objective in the convex case has been to prove Strichartz estimates. In the boundaryless case, these are a direct consequence of the dispersive estimates. However, this is not so in the boundary case. There one has the aforementioned loss, but it was apriori not at all clear whether this translates into a similar loss for the Strichartz estimates. This is because the loss in the uniform bounds is intermittent in time, so heuristically some of it may disappear in the time averaging. While a definitive result here is still elusive, Oana and her collaborators made a lot of progress in drastically narrowing the gab at both ends, improving both their counterexamples and the positive results.

The second objective of Ms. Ivanovici's work has been to study the long tome dispersive estimates in the concave case. So far this work has been carried out in the (Euclidean) exterior of a ball, with work in progress devoted to a more general case. The results here are quite surprising and also complete. Precisely, they assert that dispersive estimates hold in full up to dimension three, but exhibit a loss in higher dimension, which happens due to concentration in a (well chosen) location on the antipodal ray to the source.

To place Ms Ivanovici's results in a broader context, she took a very difficult problem (or class of problems) of considerable interest in nonlinear dispersive pde's, but for which very few results were available, and she essentially solved it nearly completely (there is still a small gap, but I am sure it won't last long). This places her right at the top in a narrow area, and among the research leaders in a broader area of pde's and microlocal analysis.

This is truly excellent work, which would certainly be more than enough to earn a tenured position in one of the leading US universities. She has demonstrated both ingeniosity, excellent technical ability, wide knowledge and perseverence, and has mastered and developed a broad array of ideas and techniques. She has her own, cohesive research program, and she is certainly capable and ready to play a leading role in the further development of the field.

I recommend in the strongest terms that Ms. Ivanovici be granted the diploma “Habilitation á diriger des recherches”, and I am looking forward to see her both her next results and her future work as a mentor and Ph.D. advisor.

Sincerely,

Daniel Tataru
Department of Mathematics
University of California, Berkeley
tataru@math.berkeley.edu
(510)-643-1284