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Letter in support of *habilitation* of Oana Ivanovici

I am writing to offer my support for the *habilitation* of Dr Oana Ivanovici. Her habilitation mémoire surveys a rich body of impressive and interesting results and indicates an extraordinary level of strength. It is beautifully illustrated by figures indicating physical orgins of the problems and the geometry involved. The theme is obtaining fine dispersion estimates for mixed boundary value problems for the wave, Klein–Gordon and Schrödinger equations.

The problems studied by Dr Ivanovici have a very long tradition firmly rooted in physics and applied mathematics. The study of effects of diffraction outside convex bodies goes back to the work of Fresnel over two hundred years ago and the effects of whispering gallery modes inside of convex bodies, to antiquity. One of the pioneers of mathematical study of diffraction, Donald Ludwig, wrote in 1969 "The asymptotic behavior of the field scattered by a convex object at high frequencies is extremely complicated" and it would be difficult in a brief review to explain all the intricacies of Dr Ivanovici's work.

There are two main directions. The first is the study the local dispersion estimates for solutions of different equations, for instance, the wave equation inside a convex domain $\Omega \subset \mathbb{R}^d$:

$$(\partial_t^2 - \Delta)u = 0, (t,x) \in \mathbb{R}_+ \times \Omega, \quad u|_{t=0} = \delta_{x_0}, \quad \partial_t u_{t=0} = 0, \quad u|_{\mathbb{R}_+ \times \partial\Omega} = 0.$$

They are of the form (see Théoreme 2)

$$|\psi(-h^2\Delta)u(t,x)| \leq Ch^{-d} \min\left(1, d(x_0, \partial\Omega)^{\frac{1}{4}}, (h/|t|)^{\frac{d-2}{2}+\frac{1}{4}} + (h/|t|)^{\frac{d-2}{2}+\frac{1}{3}}\right), \qquad (1)$$

where, $\psi \in C_c^\infty(\mathbb{R}_+)$ and $0 < h < 1$ (which should be thought of as the dyadic parameter 2^{-j} in the Littlewood–Paley decomposition). Analogues of (1) are established for Klein–Gordon equation and the Schrödinger equation and the complexity of (1) indicates the challenges faced by Dr Ivanovici (and to a much lesser, but not negligible, effect by any reviewer).

The second direction is the study of Strichartz estimates. These are closely related to estimates of the form (1) and involve $L^q([0,t]_t, L^p(\Omega))$ estimates of the solutions in terms of H^s norms of the initial data. Such estimates are central in the analysis of non-linear equations and applications in the case of diffractive boundary problems have been made by Smith–Sogge and Burq–Lebeau–Planchon, among many other leading experts. One of the most striking results surveyed in the mémoire is the counterexample (obtained in [OI6]) for the interior problem and a natural range of (p,q). What is surprising about that is the geometric origin (caustics generated in arbitrary small time near the boundary – figure on page 20) of the failure of Strichartz estimates coming from the region intermediate between very close to gliding and very far from it. Many other results on Strichartz estimates are presented. The finest ones are in the model case of the Friedlander operator. That model, based on the stratified medium wave equation has been central in the study of mixed problems since its introduction in 1976 when Friedlander proved the first microlocal propagation result in the presence of glancing.

The last topic described in the mémoire concerns precise results on diffraction by the sphere obtained in collaboration with Gilles Lebeau. What is particularly interesting is the loss in dispersive estimates at the location of the famous bright spot, predicted by Poisson based on Fresnel's theory and observed by Arago.

It is clear from this impressive list of achievements that Dr Ivanovici is capable of directing independent research. I reiterate my support for her *habilitation*.

Sincerely yours,

Maciej Zworski
Professor of Mathematics