Nonlinear Waves and Hamiltonian PDEs
La Thuile, February 20th-26th, 2022

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Schedule

	Mon	Tu	Wed	Thu	Fri
9.15 - 10.15 (Wed 9 - 10.15) (Thu 9 - 10)	Faou	Bellazzini	Berti I	Laurent	Langella
10.15 - 10.45 (Thu 10 - 10.30)	Coffee break				
10.45 - 12 (Wed 10.45 - 11.35) (Thu 10.30 - 11.20)	Vega I	Vega II	Planchon	Banica	Berti II
(Wed 11.35 - 12.25) (Thu 11.20 - 12.10)			Bambusi	Niu	
12 - 14	Free discussions				
14 - 16					
17.10 - 18 (Thu 17 - 17.50)	Cossetti	Lafontaine		Bronsard	Iandoli
18.10 - 19 (Thu 17.50 - 18.40)	Audiard	Maierhofer		Visciglia	Fanelli
(Thu 18.40 - 19.10)				Prinari	

Low regularity approximations: a decorated tree series analysis

Yvonne Alama Bronsard

In this talk I will discuss new time discretizations for solving a general class of nonlinear dispersive equations in low-regularity spaces. First, using new techniques based on decorated trees series analysis inspired by singular SPDEs, I will present a general framework for deriving low-regularity schemes up to arbitrary order. I will illustrate this decorated tree formalism for the approximation of the Gross-Pitaevskii equation.

Secondly, I will present a recent error analysis result for solving the Gross-Pitaevskii equation at low-regularity, and show first and second order convergence in any fractional positive Sobolev space H^r, $r \geq 0$. These new schemes, together with their optimal local error, allow for convergence under lower regularity assumptions than required by classical methods.

Modulational stability of periodic waves for the Schrödinger equation

Corentin Audiard

We consider the nonlinear Schrödinger equation in arbitrary dimension and study the stability of periodic waves that remain bounded away from 0. The stability is studied with respect to perturbations localized in space, for which there is roughly speaking more degrees of freedom than co-periodic stability. We introduce an Evans function, the zeros of which give the spectrum of the linearized problem. A first result is a Taylor expansion of this function close to 0, for small Bloch-Fourier parameters. This expansion can be related by formal considerations to the so-called Witham equations. The second main result is a computation of eigenfunctions close to the generalized kernel (in some sense); this allows to give an accurate description of long time dynamics for the semi group associated to the linearized problem. The results are true for a class of more general quasi-linear Schrödinger equations, with no meaningful modification of proof. Joint work with Miguel Rodrigues (IRMAR).

Microlocal analysis of singular measures

Valeria Banica

In this talk I shall present a study of scalar and vectorial measures from a microlocal point of view, by introducing a notion of L^1-regularity wave front set. The main result, based on this notion, is a characterization of the suport and polarization of the singular part of a temperate Radon measure. As a consequence we obtain a full L^1-elliptic regularity result. Also, among the consequences we obtain an extension of the result of Guido De Phillippis and Filip Rindler '16 on the properties of the singular part of measures constrained by PDEs, as well as an extension of Alberti's rank one theorem. This is a joint work with Nicolas Burq.

Finite energy traveling waves for the Gross-Pitaevskii equation in the subsonic regime

Jacopo Bellazzini

In the talk we discuss the existence of finite energy traveling waves for the Gross-Pitaevskii equation. This problem has deserved a lot of attention in the literature, but the existence of solutions in the whole subsonic range was a standing open problem till the work of Maris in 2013. However, such result is valid only in dimension 3 and higher. We prove the existence of finite energy traveling waves for almost every value of the speed in the subsonic range. Our argument works identically well in dimensions 2 and 3. Joint work with D. Ruiz (Granada).

Growth of Sobolev norms for unbounded perturbations of the Laplacian on flat tori (towards a quantum Nekhoroshev theorem)

Dario Bambusi

I will present a study of the time dependent Schrödinger equation

$$-i\psi_t = -\Delta\psi + \nu(t, x, -i\nabla)\psi$$

on a flat d dimensional torus. Here ν is a time dependent pseudo-differential operator of order strictly smaller than 2. The main result I will give is an estimate ensuring that the Sobolev norms of the solutions are bounded by t^{ϵ}. The proof is a quantization of the proof of the Nekhoroshev theorem, both analytic and geometric parts.

Previous results of this kind were limited either to the case of bounded perturbations of the Laplacian or to quantization of systems with a trivial geometry of the resonances, like harmonic oscillators or $1-d$ systems. In this seminar I will present the result and the main ideas of the proof. This is joint work with Beatrice Langella and Riccardo Montalto.

Instability of Stokes waves

Massimiliano Berti

A classical subject in fluid dynamics regards the spectral instability of Stokes waves -traveling periodic water waves-, pioneered by the famous work of Stokes in 1847. Benjamin and Feir (1967) and Zhakarov, through experiments and formal arguments, discovered that Stokes waves in deep water are unstable. More precisely, they found unstable eigenvalues near the origin of the complex plane, corresponding to small Floquet exponents μ or equivalently to long-wave perturbations. The first rigorous mathematical proofs of the local bifurcation of these eigenvalues have been given by Bridges-Mielke ('95) in finite depth and recently by Nguyen-Strauss ('20) in infinite depth. On the other hand, it has been found numerically that when the Floquet number μ varies, the above two eigenvalues trace an entire figure-eight with a cross at the origin. I will present a novel approach to prove this conjecture fully describing the unstable spectrum. The proof exploits the Hamiltonian and reversibility properties of the water waves, a symplectic version of Kato's theory of similarity transformations, and a block diagonalization idea, inspired by KAM theory. This is joint work with A. Maspero and P. Ventura.

Improved Hardy-Rellich inequalities

Lucrezia Cossetti

We investigate on Hardy-Rellich inequalities for perturbed Laplacians. We show that a non-trivial angular perturbation of the free operator typically improves the inequality, and may also provide an estimate which does not hold in the free case. The main examples are related to the introduction of a magnetic field as a manifestation of the diamagnetic phenomenon. The seminar is based on a joint work with B. Cassano and L. Fanelli.

About Schrödinger and Dirac operators with scaling critical potentials

Luca Fanelli

Lower-order perturbations of the free Hamiltonians usually appear in Quantum Mechanics, and models describing the interaction of a free particle with an external field. In some cases, the perturbation lies at the same level as the free Hamiltonian, and the resulting conflict can generate interesting phenomena. We will introduce the Inverse Square and Coulomb potentials as toy models, and describe the main features of the complete Hamiltonians from the point of view of Fourier Analysis, Spectral Theory, and dispersive evolutions. Some recent results in the setting of the Heisenberg Group will be also presented.

Around plane waves solutions of the Schrödinger-Langevin equation

Erwan Faou

We consider the logarithmic Schroedinger equations with angle damping, also called Schroedinger-Langevin equation. On a periodic domain, this equation possesses plane wave solutions that are explicit. We prove that these solutions are asymptotically stable in Sobolev regularity. In the case without damping, we prove that for almost all value of the nonlinear parameter, these solutions are stable in high Sobolev regularity for arbitrary long times when the solution is close to a plane wave. We also show and discuss numerical experiments illustrating our results. This is a joint work with Quentin Chauleur (University of Rennes).

Dispersion for the wave equation outside of a cylinder

Felice Iandoli

In this talk I will present a recent work in collaboration with Oana Ivanovici. We consider the wave equation with Dirichlet boundary conditions in the exterior of a cylinder, we construct global in time parametrix and we derive sharp dispersive estimates.

Decompositions of high-frequency Helmholtz solutions via functional calculus, and application to the finite element method

David Lafontaine

Over the last ten years, results of Melenk and Sauter decomposing high-frequency Helmholtz solutions into an analytic part and a well-behaved in frequency part have had a large impact in the numerical analysis of the Helmholtz equation. These results have been proved for the constant-coefficient Helmholtz equation outside an analytic Dirichlet obstacle. I will present our recent results obtaining analogous decompositions for scattering problems fitting into the very general black-box scattering framework of Sjöstrand and Zworski, thus covering Helmholtz problems with variable coefficients, impenetrable obstacles, and penetrable obstacles all at once. These results allow us to prove new frequency-explicit convergence results for finite-element methods applied to the Helmholtz equation.

This is a joint work with Euan Spence and Jared Wunsch.

Concentration close to the cone for linear waves

Camille Laurent

In this talk, we will be concerned with solutions to the linear wave equation. We study how the energy is asymptotically located with respect to the light cone and its translation. We derive some expressions of the exterior energy outside a shifted light cone. In particular, in odd dimension, we characterise the solutions that are asymptotically zero outside a shifted cone. This is joint work with Raphaël Côte (Strasbourg).

Growth of Sobolev norms in quasi integrable quantum systems

Beatrice Langella

In this talk I will analyze an abstract linear time dependent Schrödinger equation of the form

$$\mathrm{i}\partial_t\psi = (H_0 + V(t))\psi\,, \tag{1}$$

with H_0 a pseudo-differential operator of order $\mathtt{d} > 1$ and $V(t)$ a time dependent family of pseudo-differential operators of order strictly less than $\mathtt{d}$.
I will introduce abstract assumptions on H_0, namely steepness and global quantum integrability, under which we can prove a $|t|^\epsilon$ upper bound on the growth of Sobolev norms of all the solutions of (1).
The result I will present applies to several models, as perturbations of the quantum anharmonic oscillator in dimension 2, and perturbations of the Laplacian on a manifold with integrable geodesic flow, and in particular: flat tori, Zoll manifolds, rotation invariant surfaces and Lie groups. The case of several particles on a Zoll manifold, a torus or a Lie group is also covered.
The proof is based a on quantum version of the proof of the classical Nekhoroshev theorem.
This is a joint work with Dario Bambusi.

Analysis of oversampled collocation methods for Fredholm integral equations

Georg Maierhofer

Collocation methods for boundary integral formulations of partial differential equations are simpler and cheaper to implement than Galerkin methods because the elements of the discretisation matrix are given by lower-dimensional integrals. However, in general, their application is a delicate matter: in contrast to Galerkin methods, there is no standardised convergence theory for collocation methods, and their success is highly sensitive to the choice of collocation points. Moreover, in the integral equation setting, collocation methods typically lead to slower convergence rates than Galerkin methods.

In this talk, we will explore the extent to which the convergence properties of collocation methods for Fredholm integral equations can be improved by least-squares oversampling. We provide rigorous analysis to show that superlinear oversampling can enhance the convergence rates of the collocation method and reduce its sensitivity to the distribution of collocation points. In addition, we prove that linear oversampling can still lead to a substantial improvement in the error constant, even though the asymptotic convergence rate is not improved. Indeed, we will see an example where oversampling by a constant factor leads to an improvement of the error at a cubic rate in this constant, whilst incurring only a linear increase in cost. We support our analysis with several numerical examples for the two-dimensional Laplace and Helmholtz equation. This is joint work with Daan Huybrechs (KU Leuven).

The controllability of a special class of coupled wave system

Jingrui Niu

Let Ω be a smooth bounded domain of $\mathbb{R}^d$, $d \in \mathbb{N}^*$ and $\omega \subset \Omega$ be a subdomain. Let $\Box_1 = \partial_t^2 - d_1\Delta$ and $\Box_2 = \partial_t^2 - d_2\Delta$ be two d'Alembert operators with different constant speeds $d_1 \neq d_2$. We consider the internal controllability problem for a coupled wave system with different speeds as follows:

$$
\begin{cases}
\Box_1 U_1 + A_1 U_2 & = 0 & \text{in } (0,T)\times\Omega,\\
\Box_2 U_2 + A_2 U_2 & = bf\mathbb{1}_\omega & \text{in } (0,T)\times\Omega,\\
U_1 = U_2 & = 0 & \text{on } (0,T)\times\partial\Omega,\\
(U_1, U_2)|_{t=0} & = (U_1^0, U_2^0) & \text{in } \Omega,\\
(\partial_t U_1, \partial_t U_2)|_{t=0} & = (U_1^1, U_2^1) & \text{in } \Omega.
\end{cases}
\tag{1}
$$

For $j = 1, 2$, we use U_j to denote the solutions corresponding to the speed d_j. $f \in L^2((0,T)\times\omega)$ is the control function, which is a scalar control and acts on $(0,T)\times\omega$. $A_1 \in \mathcal{M}_{n_1,n_2}(\mathbb{R})$ and $A_2 \in \mathcal{M}_{n_2}(\mathbb{R})$ are two given coupling matrices and $b \in \mathbb{R}^{n_2}$, with two integers n_1 and n_2. Under this basic setting, we first identify the state spaces for this control problem with some compatibility conditions. Then we prove the exact controllability with the operator Kalman conditions and geometric control conditions.

This is a project in progress in collaboration with Pierre Lissy.

Bilinear estimates and growth of Sobolev Norms for 2d NLS with harmonic potential

Fabrice Planchon

We prove polynomial upper bounds on the growth of solutions to 2d cubic NLS where the Laplacian is confined by the harmonic potential, using modified energy methods. Our bounds improve on those available for the 2d cubic NLS in the periodic setting, due to better bilinear estimates for the (linear) equation. We provide an direct proof, based on integration by parts (and earlier work of Planchon-Vega), of these bilinear estimates associated with the harmonic oscillator, recovering results by A. Poiret that were derived using spectral methods and asymptotics of Bessel functions. Such arguments may be of independent interest. The talk is based on joint work with N. Visciglia and N. Tzvetkov.

A comparison principle for the Lane-Emden equation and applications

Francesca Prinari

We prove a comparison principle for positive supersolutions and subsolutions to the Lane-Emden equation for the $p-$Laplacian, with subhomogeneous power in the right-hand side. The proof uses variational tools and the result applies with no regularity assumptions, both on the set and the functions. We then show that such a comparison principle can be applied to prove: uniqueness of solutions; sharp pointwise estimates for positive solutions in convex sets; localization estimates for maximum points and sharp geometric estimates for generalized principal frequencies in convex sets.

New Conservation Laws and Energy Cascade for 1d Cubic NLS

Luis Vega

I'll present some recent results concerning the IVP of 1d cubic NLS at the critical level of regularity. I'll also exhibit a cascade of energy for the 1D Schrödinger map which is related to NLS through the so called Hasimoto transformation. For higher regularity these two equations are completely integrable systems and therefore no cascade of energy is possible. If time permits I will also present some results on the fluctuations of δ-moments for solutions of the linear Schrödinger equation.

Modified energies for the gKdV equation and applications

Nicola Visciglia

We discuss the construction of suitable energies associated with the gKdV. As a consequence we get new upper bounds on the growth of Sobolev norms as well as the quasi-invariance of the transported Gaussian measures, along with the L^p regularity of the corresponding Radon-Nykodim derivative. The talk is based on joint work with F. Planchon and N. Tzvetkov.
